Differential Geometry of Curves and Surfaces

Homework 2

Due on September 27

1. Let $\mathbf{c}(t)$ be a regular space curve with nonvanishing curvature, and let $\mathbf{c}(s) = \mathbf{c}(t(s))$ be the same curve parameterized by its arclength. Let us denote Frenet-Serret frame by $\{\mathbf{e}_1(s), \mathbf{e}_2(s), \mathbf{e}_3(s)\}$, the curvature by $k(s)$, the torsion by $\tau(s)$, the derivative with respect to t by a dot, and the derivative with respect to s by a prime. Show that:

   (a) $\dot{\mathbf{c}}(t) = \dot{s}(t)\mathbf{e}_1(s)$.

   (b) $\ddot{\mathbf{c}}(t) = \ddot{s}(t)\mathbf{e}_1(s) + \dot{s}^2(t)k(s)\mathbf{e}_2(s)$.

   (c) $\dddot{\mathbf{c}}(t) = \left(\dddot{s}(t) - \dot{s}^3(t)k^2(s)\right)\mathbf{e}_1(s) + \left(3\dot{s}(t)\ddot{s}(t)k(s) + \dot{s}^3(t)k'(s)\right)\mathbf{e}_2(s) + \dot{s}^3(t)k(s)\tau(s)\mathbf{e}_3(s)$.

   (d) $k(s) = \dfrac{\|\dot{\mathbf{c}}(t) \times \ddot{\mathbf{c}}(t)\|}{\|\dot{\mathbf{c}}(t)\|^3}$.

   (e) $\tau(s) = \dfrac{\dot{\mathbf{c}}(t) \cdot (\ddot{\mathbf{c}}(t) \times \dddot{\mathbf{c}}(t))}{\|\dot{\mathbf{c}}(t) \times \ddot{\mathbf{c}}(t)\|^2}$.