Differential Geometry of Curves and Surfaces

Homework 1

Due on September 20

1. Consider the regular plane curve $\mathbf{c} : \mathbb{R} \to \mathbb{R}^2$ given by

$$\mathbf{c}(t) = (e^t \cos(t), e^t \sin(t)),$$

parameterizing a **logarithmic spiral**.

(a) Determine the arclength function

$$s(t) = \int_0^t \|\dot{\mathbf{c}}(u)\| du,$$

and compute

$$\lim_{t \to -\infty} s(t).$$

(b) Find the curvature function for this curve.

Use a symbolic computation system, such as Mathematica, to solve the next problem

2. Consider the regular closed plane curve $\mathbf{c} : [0, 2\pi] \to \mathbb{R}^2$ given by

$$\mathbf{c}(t) = (3\cos(t) + 2\cos(2t), 3\sin(t) + 2\sin(2t)).$$

(a) Plot the curvature of this curve as well as its derivative. Based on these plots, how many vertices does the curve have?

(b) Plot the curve itself and explain why it does not violate the (general version of the) Four Vertex Theorem.